

Optical and Magnetic Studies of $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ Based on the Refined Crystal Structure

Takeshi ASAI and Ryôiti KIRIYAMA

Institute of Scientific and Industrial Research, Osaka University, Yamadakami, Suita, Osaka 565

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Magnetic, IR and electronic spectral studies were made on $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$, its crystal structure being refined. The IR band at 927 cm^{-1} was found to be due to a rocking vibration of a water molecule. The H-bonding scheme was suggested. The electronic spectrum shows a maximum of the d-d transition bands at *ca.* 14000 cm^{-1} . This was explained on the basis of a square pyramidal coordination of a Cu ion. Above 100 K the magnetic susceptibility obeys the Curie-Weiss law with $\theta = -92.6\text{ K}$ and $\mu_{\text{eff}} = 2.00\beta_{\text{B}}$. At lower temperatures it shows a broad maximum at *ca.* 45 K, a sharp peak at 26.4 K corresponding to T_{N} . The behaviour could be explained from the anti-ferromagnetic superexchange interactions of Cu cations through the σ -bonds of SeO_3^{2-} anions. The small anisotropy and the number of cations intermediated by each anion give rise to the 1-dimensional character. A linear Heisenberg model and the molecular field model gave J/k values of -33 and -31 K , respectively. The latter model also gave the observed ratio of $|\theta|/T_{\text{N}}$.

Magnetic properties of transition metal compounds have attracted the attention of many workers and studies have been carried out according to several different methods. The compounds studied can be classified into (a) those with highly anisotropic structures of a linear chain or a sheet arrangement of metal ions since the theories are expected to be phenomenologically applicable and (b) those in which the simple anions like halide or oxide anions coordinate to a metal atom since the quantum mechanics is useful for an explanation of the mechanism. They have given information on the nature of the chemical bond and the superexchange interaction. Studies on the compounds which belong to neither (a) or (b) do not seem to have been carried out. They are desirable because of their utility for testing the theories. The present study has been carried out with $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$, the easiest system for application to the theory. The trigonal-pyramidal oxo-anion SeO_3^{2-} is potential to form MO's of both a σ - and a π -type character with metal cations. However, the nature of the bonds and the mechanism of magnetic interactions depend to a great extent on the conformation attained in the crystal lattice.

Experimental

Preparation. The hydrated sample was obtained by the usual method¹⁾ and was confirmed by powder X-ray diffractometry²⁾ and also by chemical analysis of Cu content. A deuterated sample for IR study was obtained in a sealed tube at 120°C with fully dried CuSO_4 and Na_2SeO_3 dissolved into D_2O . The powder X-ray diffraction pattern of the deuterate was the same as that of the hydrate.

IR and Electronic Spectra. IR spectra were recorded in the region $4000\text{--}400\text{ cm}^{-1}$ on a JASCO DS-402G spectrometer. Both KBr pellets and nujol mulls were employed. Diffuse reflectance spectra were obtained in the range $8250\text{--}35000\text{ cm}^{-1}$ with Hitachi EPU-2 and EPS-3T spectrometers with MgO as a standard. The spectra were recorded on a pure sample and one diluted with MgO to 80 wt.-%. Dilution was confirmed to have no effect on the position of absorption maxima.

Magnetic Measurement.

Magnetic susceptibility was measured on powder samples by a Faraday balance. Temperatures were varied from 4 K to room temperature. $\text{CoHg}(\text{SCN})_4$ was used as a reference substance of susceptibility.³⁾ Corrections were made for diamagnetism⁴⁾ but not for TIP.

Refinement of the Crystal Structure. The space group and lattice constants of Gattow²⁾ were confirmed; the space group is $D_{2h}^2\text{-}P2_12_12_1$ (No. 19) and lattice constants are $a = 6.664$, $b = 9.156$ and $c = 7.369(\pm 0.005)\text{ Å}$ with $Z = 4$. Intensity data were collected using a Rigaku Denki four-circle X-ray diffractometer with a scintillation counter and $\text{MoK}\alpha$ radiation. A total of 2311 reflections in two octants were measured within $\sin \theta/\lambda$ of 0.85. By averaging intensities of symmetry-related reflections in two octants,⁵⁾ intensities for 1347 independent reflections were obtained, of which 75 reflections had less than 2σ , the e.s.d. of $|F_o|$, and were taken to be zero. Intensities were corrected for Lorentz and polarization effects, but not for absorption and extinction effects. Dimensions of the crystal were $0.05 \times 0.08 \times 0.12\text{ mm}^3$; $\mu R = 1.19$.

Using atomic parameters of Gattow's analysis²⁾ and isotropic temperature factors of 1.0 for all atoms⁶⁾ as initial parameters, four cycles of the least-squares refinements were carried out. Anisotropic temperature factors were then introduced and four more cycles were repeated. Shifts of atomic parameters and temperature factors in the last cycle were less than 0.1σ and 0.2σ , respectively, with the exception of y -parameter of $\text{O}_{\text{w}1}(0.14\sigma)$ and a temperature factor $B/2$ of Cu (0.52σ). The final R -value ($R = \sum ||F_o| - |F_c|| / \sum |F_o|$) was 0.045 (0.038 excluding $|F_o| = 0$). In the earlier stages the weights were 1.0 for all reflections. In the later stages the following weights were used.

If $|F_o| \leq 8.9$, $w = 0.25$,
if $|F_o| > 83.3$, $w = 1/|F_o|$,
and otherwise, $w = 1.0$.

3) B. N. Figgis and R. S. Nyholm, *J. Chem. Soc.*, **1958**, 4190.

4) P. W. Selwood, "Magnetochemistry," 2nd Ed., Interscience Publishers, New York (1956).

5) Since this space group lacks the center of symmetry, 2311 reflections were taken to be independent at first and refinements were performed including imaginary parts of the anomalous dispersion. The atomic parameters for the two sets of indexing were the same within the e. s. d., and their R values were 0.080 and 0.068. The difference in the intensities of I_{hkl} and $I_{h\bar{k}l}$ were therefore taken to be an error in measurement and the average values of I_{hkl} and $I_{h\bar{k}l}$ were used for the later work, which is reported here.

6) Only the atomic parameters were taken because of negative temperature factors.

1) Gmelins Handbuch der Anorganischen Chemie, Nr. 60, Cu, **B1**, 609 (1958).

2) G. Gattow, *Acta Crystallogr.*, **11**, 377 (1958).

Atomic scattering amplitudes used in these calculations were those of Cu^+ , Se and O^{2-} .⁷⁾ Corrections for the real parts of the anomalous dispersion were included. All calculations were performed with the aid of UNICS PROGRAM HBLS-IV and RSDA-4 on NEAC 2200/500 and 700 computers at the Computer Center, Osaka University.

Results and Discussion

IR Spectra. IR spectra of $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ and the dideuterate are shown in Fig. 1. The spectrum of the hydrate is in good agreement with that published by Sathianandan *et al.*⁸⁾ They assigned the band at 927 cm^{-1} to a bending vibration of Cu—O—Se. Since this band disappeared on deuteration, hydrogen atoms should take part in it. It can be assigned to a rocking vibrational mode of a water molecule. The corresponding band of a deuterate is not obvious because of its overlap with Se—O stretching bands. If we take a new band at 659 cm^{-1} as a rocking mode of D_2O , it gives 0.71 for the frequency ratio $\nu(\text{D}_2\text{O})/\nu(\text{H}_2\text{O})$, which is comparable to the observed ratios of $\nu(\text{OD})/\nu(\text{OH})$ and $\delta(\text{D}_2\text{O})/\delta(\text{H}_2\text{O})$ (0.73—0.74). Since the rocking vibration of a water of crystallization often appears at $800\text{—}700\text{ cm}^{-1}$,⁹⁾ this is a rare example of its appearing at a higher frequency.

Electronic Spectrum. The spectrum shown in Fig. 2 presents a broad *d-d* transition band at *ca.* 14000 cm^{-1} and an intense charge transfer band above 25000 cm^{-1} . The *d-d* transition band has a maximum at 13500 cm^{-1} and a shoulder at 10500 cm^{-1} . It is resolved into three components if we assume a Gaussian line shape with a common half width (Table 1).

A copper(II) cation is in an almost regular square pyramid composed of five oxygen atoms with a slight

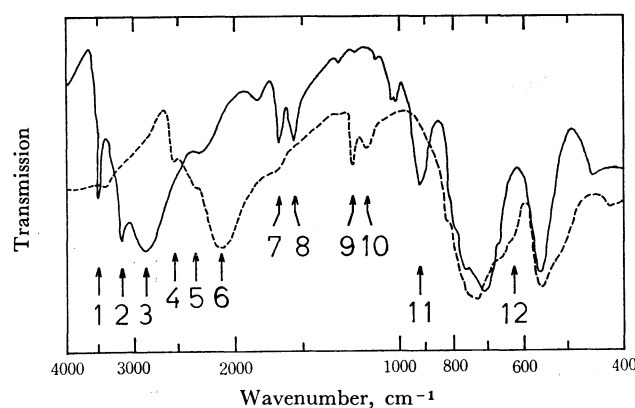


Fig. 1. Infrared spectra of $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ (—) and $\text{CuSeO}_3 \cdot 2\text{D}_2\text{O}$ (----). Bands attached with numbers are the vibrational modes involving H or D atoms.

1: ν_{OH} (3490 cm^{-1}), 2: ν_{OH} (3175 cm^{-1}), 3: ν_{OH} (2880 cm^{-1}), 4: ν_{OD} (2540 cm^{-1}), 5: ν_{OD} (2350 cm^{-1}), 6: ν_{OD} (2100 cm^{-1}), 7: δ_{HOH} (1650 cm^{-1}), 8: δ_{HOH} (1548 cm^{-1}), 9: δ_{DOD} (1215 cm^{-1}), 10: δ_{DOD} (1145 cm^{-1}), 11: $\nu_{\text{R}}(\text{H}_2\text{O})$ (927 cm^{-1}), 12: $\nu_{\text{R}}(\text{D}_2\text{O})$ (659 cm^{-1})

7) IUCr, "International Tables for Crystallography," Vol. 3, Kynoch Press, Birmingham (1962).

8) K. Sathianandan, L. D. McCorry and J. L. Margrave, *Spectrochim. Acta*, **20**, 957 (1964).

9) H. J. Prask and H. Boutin, *J. Chem. Phys.*, **45**, 699, 3284 (1966).

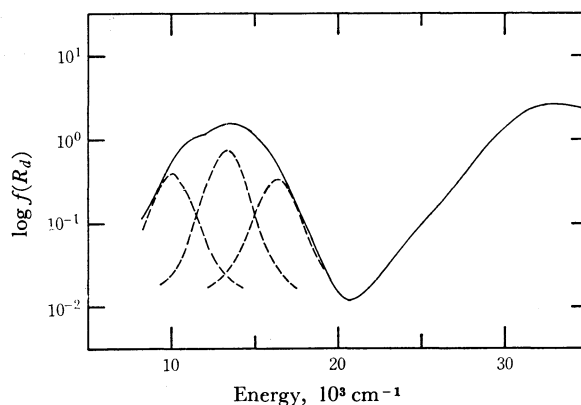


Fig. 2. Diffuse reflectance spectrum.

Observed (—) and resolved components (----).

TABLE 1. RESOLUTION AND ASSIGNMENT OF THE *d-d* TRANSITION BANDS

| Energies (cm^{-1}) | Relative intensity | Assignment ^{a)} |
|----------------------------------|-----------------------|---------------------------------------|
| 10000 | 1.4 | ${}^2A_1 \leftarrow {}^2A_2$ |
| 13200 | 1.7 | ${}^2A_1 \leftarrow {}^2A_1'$ |
| 16300 | 1.3 | ${}^2A_1 \leftarrow {}^2B_1, {}^2B_2$ |

a) Notations are given with an approximation of the C_{2v} point symmetry. A_1 and A_1' correspond to $d_{x^2-y^2}$ and d_{z^2} orbitals, respectively.

rhombic distortion (*vide post*). This configuration lacks the center of symmetry, explaining the high intensity of the absorption band. In this C_{2v} environment the $3d$ orbital levels are all split, three transitions being symmetry-allowed. The transition of $d_{x^2-y^2} \leftarrow d_{xy}$ is still forbidden, but the vibronic mechanism can bring forth the band of this transition.

Since distortion is very small, it can be said that the energy difference between d_{xz} and d_{yz} orbitals is too small to be detected in the powder reflectance spectrum. Some polarized electronic spectra¹⁰⁾ show that this difference is less than 1500 cm^{-1} and that the transitions occur at $14200\text{—}16800\text{ cm}^{-1}$. The component at 16300 cm^{-1} can therefore be assigned to the $d_{x^2-y^2} \leftarrow d_{xz}, d_{yz}$ transitions. This is also confirmed in the following estimation.

Copper(II) complexes in an environment of a tetragonally distorted octahedron of oxygen atoms give transitions of $d_{x^2-y^2} \leftarrow d_{z^2}$ in a wide range of energy. This energy was shown to be related to the axial Cu—O bond length.¹¹⁾ Smith¹²⁾ succeeded in explaining the relationship from the molecular orbital formation between copper and ligand atoms and the electrostatic crystal field energy. His treatment was modified^{12,13)} for a square pyramidal coordination with the C_{4v}

10) R. J. Dudley and B. J. Hathaway, *J. Chem. Soc. A*, **1970**, 1725; M. J. Bew, R. J. Dudley, R. J. Fereday, B. J. Hathaway and R. C. Slade, *ibid.*, **1971**, 1437; R. J. Dudley and B. J. Hathaway, *ibid.*, **1971**, 1442.

11) D. E. Billing, B. J. Hathaway and P. Nichols, *ibid.*, **1969**, 316.

12) D. W. Smith, *ibid.*, **1970**, 176.

13) C. E. Schäffer and C. K. Jørgensen, *Mol. Phys.*, **9**, 401 (1965); D. W. Smith, *J. Chem. Soc. A*, **1969**, 1708.

symmetry, giving the energies 12900, 11400 and 15100–17600 cm^{-1} for $d_{x^2-y^2} \leftarrow d_{z^2}$, $d_{x^2-y^2} \leftarrow d_{xy}$ and $d_{x^2-y^2} \leftarrow d_{xz}, d_{yz}$, respectively. These values are in good agreement with those of resolved components. π -bonding between copper and basal oxygen atoms were involved in this estimation. Since these quantities were treated only as parameters to fit the data by Smith, the agreement between resolved and estimated values does not necessarily mean the presence of the π -bonding. It might be negligible in this compound.

Refined Crystal Structure. The final atomic parameters, temperature factors and $|F_o|$'s and $|F_c|$'s are given in Tables 2, 3 and 4, respectively. The projection of the refined structure to the (100) plane is shown in Fig. 3. Interatomic distances and angles are given in Table 5.

The gross feature of the structure is the same as that of Gattow,²⁾ but significant shifts (0.06–0.15 Å) of

oxygen atoms were found. Thus the selenium and copper atoms attain more regular coordinations.

The bond lengths and angles show that the copper atom is located in the square pyramid of oxygen atoms rather than in the distorted octahedron as stated by Gattow²⁾ and Handlovič¹⁴⁾ for the isomorphous $\text{CuHPO}_3 \cdot 2\text{H}_2\text{O}$. Four oxygen atoms forming the basal plane are planar, the deviations being within 0.030 Å. The copper atom is 0.137 Å above this plane as is often the case in the copper complexes of a square pyramidal configuration. Three interatomic distances (mean 1.952 Å) of basal $\text{Cu}-\text{O}(\text{SeO}_3)$ bonds are the same within the e.s.d., but the basal $\text{Cu}-\text{O}_w1$ distance is significantly longer. The apical $\text{Cu}-\text{O}_w2$ distance became longer than that of Gattow's structure (2.27→2.323 Å), whereas the next-nearest oxygen atom O_w2' came closer than before (3.21→3.117 Å). In spite of the shortened $\text{Cu}-\text{O}_w2'$ distance this oxygen atom should be excluded from coordination to the copper atom, not only because it is much longer than the longest one in the square pyramid but also because the copper atom has three next-nearest neighbours of uncoordinating Se atoms at 3.128–3.182 Å. This configuration is in good agreement with that of $\text{CuHPO}_3 \cdot 2\text{H}_2\text{O}$,¹⁴⁾ $\text{Cu}(\text{NH}_3)_4\text{SO}_4 \cdot \text{H}_2\text{O}$ ¹⁵⁾ and $\text{Cu}(\text{NO}_3)_2 \cdot \text{CH}_3\text{NO}_2$.¹⁶⁾

A selenite anion forms a very slightly distorted trigonal pyramid. Its average dimensions are $\text{Se}-\text{O}=1.70_6$, $\text{O}-\text{O}=2.60_7$ Å and $\text{O}-\text{Se}-\text{O}=99.7^\circ$ with the selenium atom 0.80₃ Å above the basal plane. These values are comparable to those of $\text{ZnSeO}_3 \cdot 2\text{H}_2\text{O}$,¹⁷⁾ $\text{MgSeO}_3 \cdot 6\text{H}_2\text{O}$,¹⁸⁾ H_2SeO_3 ,¹⁹⁾ $\text{Na}_3\text{H}(\text{SeO}_3)_2$ ²⁰⁾ and $\text{K}_3\text{H}(\text{SeO}_3)_2$.²¹⁾ A selenite anion coordinates to three different copper atoms, with each oxygen atom towards a single copper atom, and connects the CuO_5 pyramids into a three-dimensional network.

No hydrogen atoms could be found in the analysis. However, consideration of interatomic distances and angles around O_w1 and O_w2 led to the sole probable scheme of hydrogen bonds of water molecules. A water molecule H_2O 1 may direct its hydrogen atoms towards O 1 and O 3 belonging to the two neighbouring CuO_5 pyramids and its negative dipole towards the copper atom (Table 5). The water molecule H_2O 2 has two copper neighbours. The angle $\text{Cu}-\text{O}_w2-\text{Cu}'$ and also an electrostatic repulsion between hydrogen and copper atoms suggest a configuration in which two lone pairs direct towards two copper atoms and one hydrogen atom to O 2 atom belonging to another CuO_5 neighbour. Another hydrogen atom may be free from the hydrogen bonding or incorporated to

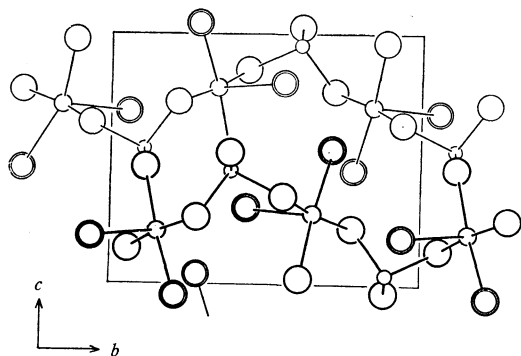


Fig. 3. Crystal structure of $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ projected to the (100) plane. The middle circle shows a Cu atom, the smallest one Se, the largest single one $\text{O}(\text{SeO}_3)$ and the largest double one $\text{O}(\text{H}_2\text{O})$.

TABLE 2. POSITIONAL PARAMETERS
Values in parentheses are the e.s.d.'s in the last decimal.

| Atom | x/a | y/b | z/c |
|----------------|-----------|-----------|-----------|
| Cu | 0.4809(1) | 0.6542(1) | 0.2860(1) |
| Se | 0.7287(1) | 0.3911(1) | 0.4573(1) |
| O 1 | 0.2556(7) | 0.7812(5) | 0.2283(6) |
| O 2 | 0.5288(6) | 0.6137(5) | 0.0295(6) |
| O 3 | 0.7335(7) | 0.5594(5) | 0.3561(6) |
| O_w 1 | 0.4489(7) | 0.7155(5) | 0.5425(7) |
| O_w 2 | 0.2921(8) | 0.4415(6) | 0.3100(8) |

TABLE 3. TEMPERATURE FACTORS ($\times 10^4$) IN THE FORM OF
 $\exp \{-(B_{11}h^2 + B_{22}k^2 + B_{33}l^2 + B_{12}hk + B_{13}hl + B_{23}kl)\}$
Values in parentheses are the e.s.d.'s in the last decimal.

| Atom | B_{11} | B_{22} | B_{33} | B_{12} | B_{13} | B_{23} |
|----------------|----------|----------|----------|----------|----------|----------|
| Cu | 55(1) | 30(1) | 34(1) | 22(2) | 0(2) | -2(1) |
| Se | 42(1) | 24(0) | 29(1) | 0(1) | 2(1) | -2(1) |
| O 1 | 64(8) | 41(4) | 52(6) | 39(10) | 46(3) | 28(8) |
| O 2 | 57(7) | 45(4) | 37(5) | 7(11) | -1(12) | -5(9) |
| O 3 | 64(8) | 29(4) | 64(6) | 11(10) | -25(13) | 7(8) |
| O_w 1 | 92(9) | 53(5) | 47(6) | 41(11) | -5(15) | -10(10) |
| O_w 2 | 94(10) | 66(8) | 85(8) | 30(13) | 9(17) | 0(12) |

- 14) M. Handlovič, *Acta Crystallogr.*, **B25**, 227 (1969).
- 15) B. Morosin, *ibid.*, **B25**, 19 (1969).
- 16) B. Duffin and S. C. Wallwork, *ibid.*, **20**, 210 (1966).
- 17) V. F. Gladkova and Yu. D. Kondrashev, *Kristallografiya*, **9**, 190 (1964).
- 18) R. Weiss, J.-P. Wendling and D. Grandjean, *Acta Crystallogr.*, **20**, 563 (1966).
- 19) F. K. Larsen, M. S. Lehmann and I. Sötofte, *Acta Chem. Scand.*, **25**, 1233 (1971).
- 20) S. F. Kaplan, M. I. Kay and B. Morosin, *Ferroelectrics*, **1**, 31 (1970).
- 21) M. S. Lehmann and F. K. Larsen, *Acta Chem. Scand.*, **25**, 3859 (1971).

TABLE 4. STRUCTURE FACTORS $3|F_o|$ AND $3|F_c|$

| K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | K | F0 | Fc | | |
|-------|-------|-----|--------|-------|-----|-------|-------|-----|--------|-------|-----|-------|-------|-------|--------|-------|-----|-------|-------|-----|-------|-------|-----|--------|-------|-----|----|----|----|--|--|
| H = 0 | L = 0 | | 6 | 152 | 149 | 1 | 256 | 259 | 2 | 43 | 41 | 7 | 33 | 31 | 3 | 56 | 50 | 3 | 96 | 99 | 8 | 116 | 116 | 10 | 68 | 74 | | | | | |
| 2 | 70 | 59 | 7 | 177 | 73 | 2 | 351 | 356 | 3 | 101 | 94 | 8 | 172 | 172 | 4 | 40 | 32 | 4 | 87 | 93 | 9 | 113 | 116 | 11 | 74 | 79 | | | | | |
| 4 | 436 | 439 | 8 | 43 | 41 | 3 | 156 | 154 | 4 | 124 | 119 | 9 | 123 | 128 | 5 | 36 | 38 | 5 | 70 | 69 | 10 | 85 | 86 | | | | | | | | |
| 6 | 50 | 41 | 9 | 31 | 25 | 4 | 156 | 158 | 5 | 0 | 18 | 10 | 70 | 70 | 6 | 50 | 52 | 6 | 119 | 118 | 11 | 37 | 39 | H = 8 | L = 4 | | | | | | |
| 8 | 127 | 122 | 10 | 113 | 108 | 5 | 89 | 89 | 6 | 52 | 50 | 11 | 54 | 50 | 7 | 96 | 97 | 7 | 96 | 97 | 12 | 43 | 46 | 0 | 62 | 58 | | | | | |
| 10 | 16 | 18 | 11 | 36 | 34 | 6 | 117 | 114 | 7 | 77 | 77 | 12 | 87 | 88 | H = 11 | L = 2 | | 8 | 98 | 101 | 13 | 37 | 41 | 1 | 21 | 19 | | | | | |
| 12 | 40 | 21 | 12 | 60 | 58 | 7 | 162 | 161 | 8 | 81 | 83 | 13 | 78 | 83 | 0 | 41 | 39 | 9 | 100 | 100 | 14 | 68 | 70 | 2 | 135 | 141 | | | | | |
| 14 | 40 | 38 | 13 | 22 | 22 | 8 | 88 | 86 | 9 | 27 | 17 | 14 | 53 | 52 | 1 | 49 | 44 | 10 | 95 | 96 | | | | 3 | 48 | 47 | | | | | |
| | | | | | | 9 | 35 | 39 | 10 | 57 | 59 | | | | 2 | 41 | 46 | 11 | 68 | 73 | H = 2 | L = 4 | | | | | | | | | |
| H = 1 | L = 0 | | H = 7 | L = 0 | | 10 | 28 | 29 | | | | H = 4 | L = 2 | | | | | 12 | 70 | 70 | 0 | 60 | 58 | 5 | 55 | 56 | | | | | |
| 1 | 300 | 294 | 1 | 144 | 137 | 11 | 96 | 95 | H = 9 | L = 1 | 0 | 0 | 6 | | | | 13 | 64 | 63 | | 1 | 239 | 242 | 6 | 70 | 69 | | | | | |
| 2 | 345 | 344 | 2 | 199 | 205 | 12 | 39 | 35 | 0 | 85 | 85 | 1 | 201 | 207 | 1 | 324 | 244 | | | | 2 | 43 | 41 | 7 | 55 | 57 | | | | | |
| 3 | 156 | 147 | 3 | 132 | 125 | 13 | 52 | 49 | 1 | 111 | 115 | 2 | 103 | 105 | 2 | 320 | 331 | H = 6 | L = 3 | | 3 | 271 | 280 | 8 | 25 | 20 | | | | | |
| 4 | 218 | 219 | 4 | 0 | 1 | 14 | 59 | 56 | 2 | 65 | 63 | 3 | 196 | 197 | 3 | 294 | 294 | 0 | 117 | 121 | 4 | 76 | 76 | 9 | 63 | 60 | | | | | |
| 5 | 238 | 240 | 5 | 46 | 46 | 15 | 30 | 27 | 3 | 48 | 46 | 4 | 80 | 82 | 4 | 79 | 80 | 1 | 133 | 132 | 5 | 158 | 157 | | | | | | | | |
| 6 | 267 | 270 | 6 | 124 | 116 | | | | 4 | 42 | 40 | 5 | 114 | 118 | 5 | 206 | 204 | 2 | 19 | 15 | 6 | 117 | 120 | H = 9 | L = 5 | | | | | | |
| 7 | 29 | 28 | 7 | 29 | 28 | | | | 5 | 108 | 108 | 6 | 180 | 180 | 6 | 209 | 204 | 3 | 130 | 130 | 7 | 124 | 123 | 0 | 57 | 59 | | | | | |
| 8 | 142 | 137 | 8 | 23 | 10 | 0 | 144 | 147 | 6 | 42 | 41 | 7 | 70 | 65 | 7 | 106 | 104 | 4 | 133 | 134 | 8 | 62 | 64 | 1 | 65 | 66 | | | | | |
| 9 | 195 | 192 | 9 | 25 | 20 | 1 | 113 | 109 | 7 | 20 | 17 | 8 | 108 | 111 | 8 | 27 | 22 | 5 | 88 | 88 | 9 | 83 | 87 | 2 | 34 | 37 | | | | | |
| 10 | 15 | 16 | 10 | 31 | 22 | 2 | 193 | 187 | 8 | 33 | 16 | 9 | 31 | 32 | 9 | 41 | 40 | 6 | 47 | 50 | 10 | 110 | 111 | 3 | 71 | 79 | | | | | |
| 11 | 32 | 36 | 11 | 32 | 20 | 3 | 70 | 64 | 9 | 55 | 59 | 10 | 151 | 155 | 10 | 32 | 29 | 7 | 86 | 86 | 11 | 0 | 20 | 4 | 63 | 66 | | | | | |
| 12 | 76 | 77 | 12 | 39 | 34 | 4 | 155 | 154 | 10 | | | 11 | 21 | 10 | 11 | 28 | 27 | 8 | 95 | 95 | 12 | 48 | 51 | 5 | 43 | 49 | | | | | |
| 13 | 88 | 90 | | | | 5 | 140 | 137 | H = 10 | L = 1 | 12 | 69 | 70 | 12 | 35 | 32 | 9 | 43 | 42 | 13 | 0 | 23 | 6 | 30 | 32 | | | | | | |
| 14 | 50 | 50 | H = 8 | L = 0 | | 6 | 240 | 238 | 0 | 70 | 69 | 13 | 0 | 18 | 13 | 37 | 38 | 10 | 37 | 36 | 14 | 43 | 49 | 7 | 67 | 57 | | | | | |
| 15 | 50 | 46 | 0 | 139 | 133 | 7 | 91 | 91 | 1 | 0 | 14 | 14 | 82 | 85 | 15 | 22 | 22 | 12 | 70 | 69 | H = 3 | L = 4 | | | | | | | | | |
| | | | 1 | 0 | 9 | 8 | 112 | 109 | 2 | 32 | 32 | | | | | | | | | | 0 | 112 | 112 | H = 10 | L = 4 | | | | | | |
| H = 2 | L = 0 | | 2 | 17 | 10 | 9 | 85 | 83 | 3 | 48 | 46 | H = 5 | L = 2 | | | | | | | | 1 | 127 | 130 | 0 | 0 | 4 | | | | | |
| 0 | 145 | 127 | 3 | 82 | 78 | 10 | 112 | 110 | 4 | 65 | 65 | 0 | 76 | 77 | 0 | 200 | 194 | H = 7 | L = 3 | | 2 | 200 | 204 | 1 | 60 | 57 | | | | | |
| 1 | 144 | 125 | 4 | 90 | 87 | 11 | 80 | 80 | 5 | 36 | 30 | 1 | 166 | 167 | 1 | 125 | 128 | 1 | 136 | 116 | 3 | 109 | 112 | 2 | 33 | 39 | | | | | |
| 2 | 256 | 232 | 5 | 26 | 13 | 12 | 112 | 112 | 6 | 50 | 49 | 2 | 196 | 197 | 2 | 266 | 274 | 2 | 92 | 94 | 4 | 149 | 156 | 3 | 38 | 42 | | | | | |
| 3 | 0 | 1 | 6 | 0 | 16 | 13 | 55 | 56 | 7 | 56 | 49 | 4 | 135 | 135 | 3 | 237 | 245 | 3 | 122 | 123 | 5 | 89 | 92 | 4 | 54 | 59 | | | | | |
| 4 | 32 | 31 | 7 | 135 | 131 | 14 | 39 | 29 | H = 11 | L = 1 | 5 | 162 | 162 | 4 | 168 | 169 | 4 | 111 | 112 | 6 | 125 | 129 | 5 | 31 | 34 | | | | | | |
| 5 | 83 | 80 | 8 | 31 | 28 | | | | 0 | 0 | 17 | 6 | 140 | 136 | 5 | 51 | 50 | 5 | 99 | 97 | 7 | 95 | 98 | | | | | | | | |
| 6 | 325 | 321 | 9 | 47 | 46 | H = 4 | L = 1 | | 1 | 66 | 66 | 7 | 124 | 122 | 6 | 157 | 158 | 6 | 53 | 57 | 8 | 87 | 88 | H = 0 | L = 5 | | | | | | |
| 7 | 41 | 38 | 10 | 0 | | 1 | 123 | 122 | 2 | 23 | 17 | 8 | 38 | 32 | 7 | 170 | 172 | 7 | 55 | 55 | 9 | 86 | 85 | 1 | 69 | 73 | | | | | |
| 8 | 59 | 55 | 11 | 111 | 113 | 2 | 111 | 111 | 3 | 26 | 31 | 9 | 53 | 48 | 8 | 90 | 90 | 8 | 46 | 46 | 10 | 102 | 104 | 2 | 157 | 159 | | | | | |
| 9 | 11 | 13 | | | | 3 | 210 | 206 | H = 0 | L = 2 | 11 | 58 | 60 | 10 | 51 | 53 | 9 | 73 | 74 | 9 | 53 | 52 | 11 | 82 | 82 | 3 | 38 | 42 | | | |
| 10 | 194 | 189 | H = 9 | L = 0 | | 4 | 159 | 150 | 0 | 20 | 15 | 12 | 21 | 24 | 11 | 86 | 85 | 10 | 9 | 6 | 12 | 37 | 32 | 4 | 108 | 107 | | | | | |
| 11 | 25 | 21 | 1 | 17 | 15 | 5 | 42 | 41 | 1 | 306 | 309 | 13 | 28 | 31 | 12 | 70 | 71 | 12 | 70 | 71 | 13 | 28 | 21 | 5 | 157 | 156 | | | | | |
| 12 | 87 | 85 | 2 | 78 | 73 | 6 | 68 | 66 | 2 | 113 | 115 | | | | 13 | 61 | 61 | H = 8 | L = 3 | | 14 | 80 | 80 | 6 | 103 | 100 | | | | | |
| 13 | 19 | 9 | 3 | 38 | 40 | 7 | 188 | 188 | 3 | 116 | 118 | H = 6 | L = 2 | | 14 | 52 | 48 | 0 | 47 | 46 | H = 4 | L = 4 | | 8 | 180 | 177 | | | | | |
| 14 | 96 | 96 | 4 | 51 | 52 | 8 | 157 | 159 | 4 | 125 | 116 | 0 | 179 | 178 | 15 | 50 | 48 | 1 | 66 | 68 | 0 | 202 | 206 | 9 | 63 | 63 | | | | | |
| 15 | 23 | 6 | 5 | 43 | 46 | 9 | 25 | 26 | 4 | 125 | 116 | 1 | 43 | 44 | | | | 2 | 72 | 74 | 1 | 21 | 18 | 10 | 75 | 77 | | | | | |
| | | | 6 | 60 | 55 | 10 | 78 | 78 | 5 | 134 | 128 | 2 | 43 | 44 | 0 | 60 | 64 | 3 | 60 | 62 | 2 | 166 | 172 | 11 | 89 | 91 | | | | | |
| H = 3 | L = 0 | | 7 | 63 | 61 | 11 | 118 | 117 | 6 | 262 | 260 | 3 | 66 | 67 | 0 | 60 | 64 | 4 | 36 | 38 | 3 | 71 | 70 | 12 | 86 | 89 | | | | | |
| 1 | 287 | 276 | 8 | 92 | 92 | 12 | 70 | 70 | 7 | 38 | 37 | 4 | 159 | 157 | 1 | 96 | 97 | 5 | 78 | 79 | 4 | 128 | 130 | 13 | 0 | 1 | | | | | |
| 2 | 403 | 402 | 9 | 57 | 63 | 13 | 47 | 46 | 8 | 135 | 133 | 4 | 159 | 157 | 2 | 29 | 26 | 6 | 90 | 91 | 5 | 143 | 146 | 14 | 27 | 31 | | | | | |
| 3 | 232 | 226 | H = 10 | L = 0 | | 9 | 0 | 2 | 10 | 232 | 226 | 6 | 95 | 91 | 3 | 116 | 118 | 7 | 38 | 37 | 6 | 55 | 55 | | | | | | | | |
| 4 | 145 | 138 | 0 | 0 | 16 | H = 5 | L = 1 | | 11 | 0 | 4 | 7 | 99 | 98 | 4 | 194 | 200 | 8 | 45 | 45 | 7 | 60 | 65 | H = 1 | L = 5 | | | | | | |
| 5 | 166 | 138 | 1 | 102 | 100 | 0 | 235 | 231 | 12 | 83 | 80 | 8 | 45 | 48 | 5 | 95 | 96 | 9 | 60 | 62 | 8 | 61 | 65 | 0 | 107 | 108 | | | | | |
| 6 | 17 | 9 | 2 | 0 | 1 | 1 | 171 | 172 | 13 | 20 | 6 | 9 | 73 | 71 | 6 | 126 | 128 | 10 | 65 | 70 | 9 | 132 | 140 | 1 | 205 | 208 | | | | | |
| 7 | 53 | 53 | 3 | 75 | 72 | 2 | 214 | 205 | 14 | 112 | 113 | 10 | 25 | 21 | 7 | 147 | 150 | | | | 10 | 20 | 25 | 2 | 73 | 69 | | | | | |
| 8 | 131 | 128 | 4 | 0 | 3 | 3 | 75 | 72 | 15 | 26 | 26 | 11 | 78 | 76 | 8 | 161 | 165 | | | | 11 | 61 | 66 | 3 | 257 | 263 | | | | | |
| 9 | 15 | 16 | 5 | 84 | 81 | 4 | 121 | 120 | 16 | 126 | 126 | 12 | 28 | 30 | 9 | 66 | 65 | 0 | 45 | 48 | 12 | 0 | 4 | 4 | 120 | 120 | | | | | |
| 10 | 21 | 0 | 6 | 41 | 37 | 5 | 142 | 136 | H = 1 | L = 2 | 0 | 210 | 210 | H = 7 | L = 2 | 12 | 113 | 113 | 2 | 52 | 54 | 13 | 83 | 90 | 5 | 40 | 38 | | | | |
| 11 | 38 | 38 | | | | 6 | 142 | 137 | 1 | 260 | 260 | 0 | 112 | 107 | 12 | 133 | 134 | 3 | 24 | 25 | H = 5 | L = 4 | | 7 | 173 | 173 | | | | | |
| 12 | 42 | 37 | H = 11 | L = 0 | | 8 | 89 | 89 | 2 | 171 | 177 | 1 | 72 | 72 | 13 | 24 | 21 | 4 | 34 | 37 | 0 | 180 | 186 | 8 | 45 | 46 | | | | | |
| 13 | 54 | 40 | 1 | 42 | 38 | 9 | 89 | 89 | 3 | 96 | 96 | 2 | 38 | 36 | 14 | 46 | 45 | 5 | 70 | 73 | 1 | 141 | 145 | 9 | 76 | 73 | | | | | |
| H = 4 | L = 0 | | 3 | 44 | 46 | 10 | 80 | 85 | 4 | 153 | 155 | 3 | 69 | 70 | | | | 6 | 57 | 59 | 2 | 0 | 15 | 10 | 26 | 33 | | | | | |
| 0 | 564 | 569 | 11 | 18 | 12 | 5 | 272 | 275 | 4 | 117 | 113 | H = 3 | L = 3 | | 7 | 36 | 31 | 3 | 102 | 100 | 3 | 102 | 100 | 11 | 91 | 89 | | | | | |
| 1 | 28 | 33 | H = 0 | L = 1 | | 12 | 70 | 67 | 6 | 135 | 134 | 5 | 76 | 74 | 0 | 218 | 221 | 8 | 50 | 58 | 4 | 154 | 155 | 12 | 21 | 18 | | | | | |
| 2 | 23 | 27 | 1 | 143 | 146 | 7 | 82</ | | | | | | | | | | | | | | | | | | | | | | | | |

TABLE 4. (Continued)

| K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC | K | FO | FC |
|-------------|--------------|-------------|-------------|-------------|-------------|--------------|-------------|--------------|-------------|--------------|--------------|-------------|--------------|--------------|-----------|-----------|-------------|----------|----------|---------|----------|----------|----------|----------|----|----|---|----|----|
| H = 4 L = 5 | 6 | 39 | 36 | 3 | 167 | 171 | 8 | 85 | 83 | H = 6 L = 7 | 10 | 50 | 49 | 7 | 41 | 45 | H = 7 L = 9 | 4 | 57 | 61 | | | | | | | | | |
| 0 83 81 | 7 | 0 | 20 | 4 | 43 | 45 | 9 | 61 | 62 | 0 42 43 | 11 | 0 | 10 | 8 | 23 | 20 | 0 61 60 | | | | | | | | | | | | |
| 1 87 91 | | | | 5 | 88 | 92 | 10 | 44 | 44 | 1 108 108 | | | | 9 | 30 | 32 | 1 79 79 | | | | | | | | | | | | |
| 2 113 115 | H = 10 L = 5 | 6 | 24 | 24 | 11 | 80 | 76 | 2 85 86 | H = 3 L = 8 | 10 | 30 | 28 | 2 63 67 | | | | 2 63 67 | | | | | | | | | | | | |
| 3 54 57 | 0 42 38 | 7 | 76 | 75 | 12 | 61 | 61 | 3 57 62 | 0 74 72 | | | | 3 62 65 | | | | 4 47 48 | | | | | | | | | | | | |
| 4 113 113 | 1 44 50 | 8 | 82 | 88 | | | | 4 31 33 | 1 85 85 | H = 1 L = 9 | | | | 4 47 48 | | | | | | | | | | | | | | | |
| 5 129 129 | 2 35 34 | 9 | 41 | 41 | H = 1 L = 7 | 0 39 30 | 5 82 88 | 5 82 88 | 2 87 90 | 1 66 65 | | | | H = 0 L = 10 | 5 | 0 | 3 | | | | | | | | | | | | |
| 6 61 62 | 3 0 20 | 10 | 33 | 32 | 0 125 125 | 7 0 6 | 6 77 79 | 6 77 79 | 3 48 48 | 2 84 85 | | | | 0 73 71 | 6 | 17 | 22 | | | | | | | | | | | | |
| 7 81 82 | | 11 | 0 | 6 | 2 90 87 | 8 38 42 | 5 92 90 | 3 103 101 | 5 92 90 | 2 84 85 | | | | 7 111 104 | | | | | | | | | | | | | | | |
| 8 125 126 | H = 0 L = 6 | 12 | 54 | 53 | 3 204 202 | 9 71 74 | 6 81 81 | 4 46 43 | 2 33 32 | | | | | | | | | | | | | | | | | | | | |
| 9 56 56 | 0 0 17 | | | | 4 52 51 | | 7 68 65 | 5 29 23 | 3 85 83 | | | | | | | | | | | | | | | | | | | | |
| 10 50 55 | 1 235 238 | H = 5 L = 6 | 5 61 60 | 5 61 60 | 5 61 60 | 10 22 21 | 4 78 83 | H = 4 L = 8 | 0 79 77 | H = 2 L = 9 | | | | | | | | | | | | | | | | | | | |
| 11 66 73 | 2 55 54 | 0 65 61 | 5 20 25 | 0 84 75 | 8 42 44 | 6 87 85 | 4 53 47 | 0 72 70 | | | | | | | | | | | | | | | | | | | | | |
| 12 57 65 | 3 226 229 | 1 21 15 | 6 20 25 | 1 48 48 | 10 66 67 | 8 58 55 | 6 100 97 | 2 106 104 | | | | | | | | | | | | | | | | | | | | | |
| 13 21 19 | 4 12 11 | 2 20 12 | 7 155 156 | 2 37 37 | 11 62 65 | 10 60 43 | 8 44 43 | 4 57 57 | | | | | | | | | | | | | | | | | | | | | |
| H = 5 L = 5 | 6 33 32 | 4 28 30 | 9 53 51 | 3 68 69 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 152 152 | 7 117 114 | 5 69 67 | 10 22 21 | 4 78 83 | H = 4 L = 8 | 0 79 77 | H = 2 L = 9 | | | | | | | | | | | | | | | | | | | | | | |
| 1 127 131 | 8 0 14 | 6 93 96 | 11 76 80 | 5 38 32 | 0 79 77 | H = 2 L = 9 | | | | | | | | | | | | | | | | | | | | | | | |
| 2 108 108 | 9 48 46 | 7 111 109 | 12 0 4 | 6 42 45 | 1 22 24 | H = 1 L = 10 | | | | | | | | | | | | | | | | | | | | | | | |
| 3 150 156 | 10 0 0 | 8 48 53 | H = 2 L = 7 | 8 64 68 | 2 86 89 | 1 42 40 | 0 63 64 | H = 2 L = 11 | | | | | | | | | | | | | | | | | | | | | |
| 4 127 131 | 11 19 9 | 9 46 44 | 0 32 28 | 1 180 181 | 0 67 68 | 2 44 43 | 1 111 107 | 0 0 12 | | | | | | | | | | | | | | | | | | | | | |
| 5 41 39 | 12 0 7 | 10 98 101 | 0 172 175 | 2 172 175 | 3 88 88 | 3 69 68 | 2 53 55 | 1 106 103 | | | | | | | | | | | | | | | | | | | | | |
| 6 43 46 | 13 0 5 | 11 86 89 | 3 88 88 | 4 88 90 | 5 103 99 | 4 60 59 | 3 42 45 | 2 82 83 | | | | | | | | | | | | | | | | | | | | | |
| 7 89 91 | H = 1 L = 6 | H = 6 L = 6 | 4 88 90 | 2 40 38 | 3 32 28 | 9 75 81 | 8 84 80 | 5 58 60 | | | | | | | | | | | | | | | | | | | | | |
| 8 58 62 | 0 180 176 | 0 0 7 | 4 88 90 | 3 32 28 | 4 61 61 | 10 26 24 | 9 49 46 | 6 39 43 | | | | | | | | | | | | | | | | | | | | | |
| 9 45 40 | 1 73 73 | 1 40 40 | 5 103 99 | 6 138 136 | 7 18 10 | 5 65 73 | 6 51 51 | H = 3 L = 11 | | | | | | | | | | | | | | | | | | | | | |
| 10 42 42 | 2 29 22 | 2 161 161 | 6 138 136 | 7 18 10 | 8 69 68 | 6 51 51 | H = 5 L = 8 | 0 127 128 | H = 3 L = 9 | H = 2 L = 10 | 0 57 52 | | | | | | | | | | | | | | | | | | |
| 11 32 36 | 3 0 10 | 3 31 26 | 7 18 10 | 8 69 68 | 9 56 56 | | | 0 147 145 | 1 110 106 | 1 24 19 | 1 0 6 | | | | | | | | | | | | | | | | | | |
| 12 23 17 | 4 144 142 | 4 18 13 | 8 69 68 | 9 56 56 | 10 47 49 | H = 9 L = 7 | 0 68 76 | 2 37 33 | 1 31 28 | 2 39 43 | 3 39 43 | 4 107 103 | 3 109 105 | 4 47 44 | 5 46 47 | 6 62 64 | | | | | | | | | | | | | |
| H = 6 L = 5 | 5 133 132 | 5 88 90 | 9 56 56 | 10 47 49 | H = 3 L = 7 | 0 62 61 | H = 0 L = 8 | 0 125 123 | 9 33 44 | 10 25 17 | H = 3 L = 10 | 2 35 31 | 3 55 50 | | | | | | | | | | | | | | | | |
| 0 74 71 | 6 99 96 | 6 107 112 | 10 47 49 | H = 9 L = 7 | 0 68 76 | 2 37 33 | 1 31 28 | 2 39 43 | 3 39 43 | 4 107 103 | 3 109 105 | 4 47 44 | 5 46 47 | 6 62 64 | | | | | | | | | | | | | | | |
| 1 112 113 | 7 85 87 | 7 33 37 | 12 78 75 | H = 3 L = 7 | 0 62 61 | 1 113 114 | 2 19 19 | 0 125 123 | 9 33 44 | 10 25 17 | H = 3 L = 10 | 2 35 31 | 3 55 50 | | | | | | | | | | | | | | | | |
| 2 152 155 | 8 112 112 | 8 0 7 | 12 78 75 | H = 3 L = 7 | 0 62 61 | 1 113 114 | 2 19 19 | 0 125 123 | 9 33 44 | 10 25 17 | H = 3 L = 10 | 2 35 31 | 3 55 50 | | | | | | | | | | | | | | | | |
| 3 44 44 | 9 66 64 | 9 104 112 | H = 3 L = 7 | 0 62 61 | 1 113 114 | 2 19 19 | 0 125 123 | 9 33 44 | 10 25 17 | H = 3 L = 10 | 2 35 31 | 3 55 50 | | | | | | | | | | | | | | | | | |
| 4 37 34 | 10 120 119 | 10 27 31 | H = 3 L = 7 | 0 62 61 | 1 113 114 | 2 19 19 | 0 125 123 | 9 33 44 | 10 25 17 | H = 3 L = 10 | 2 35 31 | 3 55 50 | | | | | | | | | | | | | | | | | |
| 5 102 104 | 11 89 91 | H = 7 L = 6 | 0 145 142 | 2 19 19 | 3 158 159 | 4 37 57 | 5 51 53 | 3 84 81 | H = 6 L = 8 | 0 22 19 | H = 4 L = 9 | 0 25 16 | 1 87 87 | 4 61 60 | | | | | | | | | | | | | | | |
| 6 73 76 | 12 34 38 | H = 7 L = 6 | 0 145 142 | 2 19 19 | 3 158 159 | 4 37 57 | 5 51 53 | 3 84 81 | H = 6 L = 8 | 0 22 19 | H = 4 L = 9 | 0 25 16 | 1 87 87 | 4 61 60 | | | | | | | | | | | | | | | |
| 7 29 26 | 13 0 3 | 0 145 142 | 2 19 19 | 3 158 159 | 4 37 57 | 5 51 53 | 3 84 81 | H = 6 L = 8 | 0 22 19 | H = 4 L = 9 | 0 25 16 | 1 87 87 | 4 61 60 | | | | | | | | | | | | | | | | |
| 8 37 40 | H = 2 L = 6 | 2 26 16 | 4 37 57 | 5 51 53 | 3 84 81 | H = 6 L = 8 | 0 22 19 | H = 4 L = 9 | 0 25 16 | 1 87 87 | 4 61 60 | | | | | | | | | | | | | | | | | | |
| 9 96 102 | 0 26 26 | 2 26 16 | 4 37 57 | 5 51 53 | 3 84 81 | H = 6 L = 8 | 0 22 19 | H = 4 L = 9 | 0 25 16 | 1 87 87 | 4 61 60 | | | | | | | | | | | | | | | | | | |
| 10 58 59 | 1 87 88 | 4 112 114 | 6 32 31 | 4 92 89 | 5 140 135 | 2 29 24 | 2 29 24 | 3 81 83 | 1 95 92 | 4 67 64 | H = 5 L = 11 | 0 46 53 | 1 58 57 | | | | | | | | | | | | | | | | |
| 11 39 34 | 2 87 87 | 5 55 56 | 6 0 8 | 8 63 61 | 9 61 65 | 10 34 28 | 9 103 103 | 5 61 65 | 4 0 19 | 6 23 16 | 7 28 28 | 2 86 85 | | | | | | | | | | | | | | | | | |
| H = 7 L = 5 | 3 26 26 | 6 0 8 | 8 63 61 | 9 61 65 | 10 34 28 | 9 103 103 | 5 61 65 | 4 0 19 | 6 23 16 | 7 28 28 | 2 86 85 | | | | | | | | | | | | | | | | | | |
| 0 55 57 | 4 15 16 | 7 35 40 | 9 61 65 | 10 34 28 | 9 103 103 | 5 61 65 | 4 0 19 | 6 23 16 | 7 28 28 | 2 86 85 | | | | | | | | | | | | | | | | | | | |
| 1 56 53 | 5 164 165 | 8 54 54 | 10 34 28 | 9 103 103 | 5 61 65 | 4 0 19 | 6 23 16 | 7 28 28 | 2 86 85 | | | | | | | | | | | | | | | | | | | | |
| 2 50 50 | 6 74 70 | 9 26 11 | 11 80 80 | 12 28 28 | 11 50 48 | 8 71 72 | H = 7 L = 8 | 0 19 0 | 9 24 28 | 1 71 69 | 2 0 10 | | | | | | | | | | | | | | | | | | |
| 3 70 71 | 7 96 95 | H = 8 L = 6 | 0 0 13 | 1 56 55 | 2 113 111 | 3 25 26 | 4 73 74 | 3 56 57 | 4 131 130 | 5 129 128 | 4 37 33 | 3 39 41 | 5 38 35 | H = 1 L = 12 | 0 0 25 | | | | | | | | | | | | | | |
| 4 79 80 | 8 19 19 | H = 8 L = 6 | 0 0 13 | 1 56 55 | 2 113 111 | 3 25 26 | 4 73 74 | 3 56 57 | 4 131 130 | 5 129 128 | 4 37 33 | 3 39 41 | 5 38 35 | H = 1 L = 12 | 0 0 25 | | | | | | | | | | | | | | |
| 5 44 37 | 9 212 212 | 0 0 13 | 1 56 55 | 2 113 111 | 3 25 26 | 4 73 74 | 3 56 57 | 4 131 130 | 5 129 128 | 4 37 33 | 3 39 41 | 5 38 35 | H = 1 L = 12 | 0 0 25 | | | | | | | | | | | | | | | |
| 6 69 66 | 10 34 38 | 1 55 55 | 2 24 28 | 3 72 71 | 4 60 61 | 5 30 27 | 6 29 23 | 7 28 21 | 8 83 89 | 9 55 58 | 10 40 33 | 11 62 63 | H = 5 L = 7 | 0 95 100 | 1 77 77 | 2 98 98 | 3 130 130 | 4 78 82 | 5 29 35 | 6 47 49 | 7 90 90 | 8 41 41 | 9 31 22 | 10 37 39 | | | | | |
| 7 81 82 | 11 43 41 | 2 24 28 | 3 72 71 | 4 60 61 | 5 30 27 | 6 29 23 | 7 28 21 | 8 83 89 | 9 55 58 | 10 40 33 | 11 62 63 | H = 5 L = 7 | 0 95 100 | 1 77 77 | 2 98 98 | 3 130 130 | 4 78 82 | 5 29 35 | 6 47 49 | 7 90 90 | 8 41 41 | 9 31 22 | 10 37 39 | | | | | | |
| 8 75 79 | 12 0 16 | 3 72 71 | 4 60 61 | 5 30 27 | 6 29 23 | 7 28 21 | 8 83 89 | 9 55 58 | 10 40 33 | 11 62 63 | H = 5 L = 7 | 0 95 100 | 1 77 77 | 2 98 98 | 3 130 130 | 4 78 82 | 5 29 35 | 6 47 49 | 7 90 90 | 8 41 41 | 9 31 22 | 10 37 39 | | | | | | | |
| 9 48 51 | 13 113 115 | 4 60 61 | 5 30 27 | 6 29 23 | 7 28 21 | 8 83 89 | 9 55 58 | 10 40 33 | 11 62 63 | H = 5 L = 7 | 0 95 100 | 1 77 77 | 2 98 98 | 3 130 130 | 4 78 82 | 5 29 35 | 6 47 49 | 7 90 90 | 8 41 41 | 9 31 22 | 10 37 39 | | | | | | | | |
| 10 68 70 | H = 3 L = 6 | 6 29 23 | 7 28 21 | 8 83 89 | 9 55 58 | 10 40 33 | 11 62 63 | H = 5 L = 7 | 0 95 100 | 1 77 77 | 2 98 98 | 3 130 130 | 4 78 82 | 5 29 35 | 6 47 49 | 7 90 90 | 8 41 41 | 9 31 22 | 10 37 39 | | | | | | | | | | |
| H = 8 L = 5 | 0 205 210 | 7 28 21 | 8 83 89 | 9 55 58 | 10 40 33 | 11 62 63 | H = 5 L = 7 | 0 95 100 | 1 77 77 | 2 98 98 | 3 130 130 | 4 78 82 | 5 29 35 | 6 47 49 | 7 90 90 | 8 41 41 | 9 31 22 | 10 37 39 | | | | | | | | | | | |
| 0 79 80 | 1 144 142 | 8 83 89 | | | | | | | | | | | | | | | | | | | | | | | | | | | |

TABLE 5. INTERATOMIC DISTANCES (Å) AND ANGLES (°) WITH THEIR e.s.d.'s.
The subscript indicates symmetry operations given below to derive the atomic parameters.

| 1 | x | y | z | 5 | $x-1/2$ | $3/2-y$ | $1-z$ |
|---|---------|---------|---------|---|---------|---------|---------|
| 2 | $1/2-x$ | $1-y$ | $z+1/2$ | 6 | $1-x$ | $y-1/2$ | $1/2-z$ |
| 3 | $1/2-x$ | $1-y$ | $z-1/2$ | 7 | $1-x$ | $y-1/2$ | $3/2-z$ |
| 4 | $x+1/2$ | $3/2-y$ | $1-z$ | | | | |

| | | | | | | | |
|---|----------|---|------------|--|----------|--|------------|
| A. CuO ₅ square pyramid | | | | O 1-O 2 | 2.611(6) | O 1-O 2-O 3 | 60.70(17) |
| Cu-O 1 | 1.946(5) | O 1-Cu-O 2 | 91.61(19) | O 2-O 3 | 2.580(6) | O 2-O 3-O 1 | 60.25(17) |
| Cu-O 2 | 1.952(4) | O 2-Cu-O 3 | 91.75(19) | O 3-O 1 | 2.623(6) | O 3-O 1-O 2 | 59.05(17) |
| Cu-O 3 | 1.963(5) | O 3-Cu-O _w 1 | 88.08(20) | C. Around O _w 1 | | | |
| Cu-O _w 1 | 1.983(5) | O _w 1-Cu-O 1 | 87.49(20) | O _w 1-O 1 ₄ | 2.652(7) | O 1 ₄ -O _w 1-O 3 ₅ | 103.39(22) |
| Cu-O _w 2 | 2.325(6) | O _w 2-Cu-O 1 | 95.72(20) | O _w 1-O 3 ₅ | 2.620(7) | O 1 ₄ -O _w 1-O _w 2 ₂ | 89.97(21) |
| | | O _w 2-Cu-O 2 | 90.17(19) | O _w 1-O _w 2 ₂ | 2.921(8) | O 3 ₅ -O _w 1-O _w 2 ₂ | 83.88(21) |
| O 1-O 2 | 2.795(6) | O _w 2-Cu-O 3 | 94.22(19) | | | Cu ₁ -O _w 1-O 1 ₄ | 121.83(24) |
| O 2-O 3 | 2.811(6) | O _w 2-Cu-O _w 1 | 96.11(20) | D. Around O _w 2 | | | |
| O 3-O _w 1 | 2.743(7) | | | O _w 2-O 2 ₂ | 2.729(7) | O 2 ₂ -O _w 2-O _w 1 ₃ | 82.99(21) |
| O _w 1-O 1 | 2.717(7) | O 1-O 2-O 3 | 87.96(18) | O _w 2-O _w 1 ₃ | 2.921(8) | O 2 ₂ -O _w 2-O 1 ₆ | 132.15(23) |
| O _w 2-O 1 | 3.177(7) | O 2-O 3-O _w 1 | 90.06(20) | O _w 2-O 1 ₆ | 3.364(8) | O _w 1 ₃ -O _w 2-O 1 ₆ | 102.79(20) |
| O _w 2-O 2 | 3.041(7) | O 3-O _w 1-O 1 | 90.94(21) | | | Cu ₁ -O _w 2-O 2 ₂ | 128.63(25) |
| O _w 2-O 3 | 3.152(8) | O _w 1-O 1-O 2 | 90.94(20) | | | Cu ₁ -O _w 2-O _w 1 ₃ | 131.18(25) |
| O _w 2-O _w 1 | 3.213(8) | | | | | Cu ₁ -O _w 2-O 1 ₆ | 82.78(18) |
| Cu-O _w 2 ₇ | 3.116(6) | O _w 2 ₁ -Cu-O _w 2 ₇ | 170.76(17) | | | Cu ₆ -O _w 2-O 2 ₂ | 111.02(21) |
| B. SeO ₃ ²⁻ anion | | | | | | Cu ₆ -O _w 2-O _w 1 ₃ | 72.44(16) |
| Se-O 1 | 1.701(4) | O 1-Se-O 2 | 100.23(22) | | | Cu ₆ -O _w 2-O 1 ₆ | 34.68(10) |
| Se-O 2 | 1.702(5) | O 2-Se-O 3 | 98.14(22) | | | Cu ₁ -O _w 2-Cu ₆ | 115.28(21) |
| Se-O 3 | 1.712(4) | O 3-Se-O 1 | 100.43(21) | | | | |

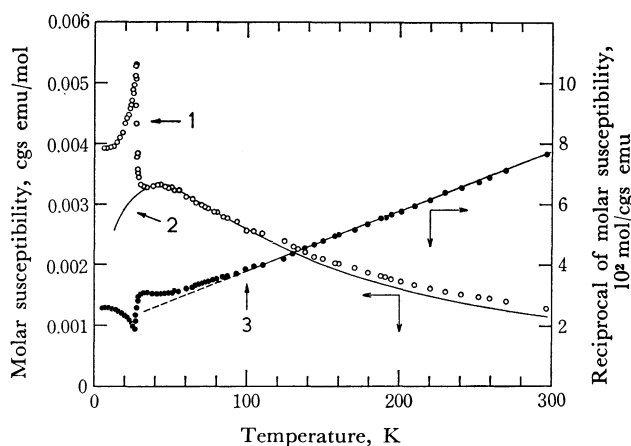


Fig. 4. Molar magnetic susceptibility and its reciprocal plotted against temperature.

1: Observed molar susceptibility, 2: calculated one for a linear Heisenberg spin system with $g=2.00$ and $J/k=-33$ K, 3: reciprocal of the observed one.

ceptibilities below T_N were observed, and in the case of $\text{Cu}(\text{HCOO})_2 \cdot 4\text{H}_2\text{O}$ ²⁵⁾ the magnetization was measured to be 3% of the moment of a Cu(II) ion originating from the canting of the spins. The magnetic susceptibility of $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ can also be explained in a similar manner.

The possible paths of superexchange interaction between magnetic ions were sought in the crystal structure. A linear path of $\text{Cu}-\text{H}_2\text{O} \cdots \text{Cu}$ along the crystallographic b axis is most apparent and a conformable

path of magnetic interaction is known for $\text{Cu}(\text{NH}_3)_4 \cdot \text{SO}_4 \cdot \text{H}_2\text{O}$.²⁸⁾ However, the interaction through an H_2O molecule is very weak as is evident from the small θ and J values found in $\text{Cu}(\text{NH}_3)_4 \cdot \text{SO}_4 \cdot \text{H}_2\text{O}$ ²⁸⁾ and $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$,²⁹⁾ and cannot explain the observed maximum at high temperatures. The other paths should involve SeO_3^{2-} anions. Each SeO_3^{2-} anion combines three different CuO_5 square pyramids and forms a three-dimensional network. Interatomic distances and angles concerning three Se-O-Cu paths are almost the same. If these paths contribute equally to the magnetic interaction, $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ should be a three-dimensional ferromagnet or antiferromagnet. As a matter of fact, a broad maximum of susceptibility characteristic of the one- or two-dimensional magnet is observed. There must, therefore, be a certain difference between the three paths of Cu-O-Se-O-Cu. The environments of three oxygen atoms of an SeO_3^{2-} anion are schematically projected along the pseudosymmetry axis of the anion in Fig. 5. These oxygen atoms are also a proton acceptor. Bond angles which are made by Se, Cu and a hydrogen-donor atoms are very close to 120° except $\text{O}_{w14}-\text{O}_{32}-\text{Cu}_2$. These values suggest that the orbitals of oxygen atoms are the sp^2 hybrid ones and that one of them is used for a σ -bond with the Se atom while the other two lone pairs are

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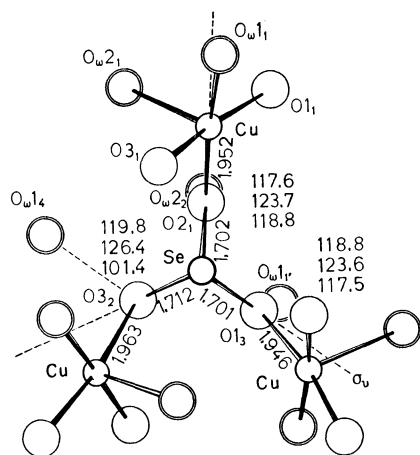


Fig. 5. Environment around an SeO_3^{2-} anion viewed along the pseudo-symmetry axis of the anion. Three numbers given for each oxygen atom of the anion are the angles of Se-O-Cu, Se-O-O(hydrogen donor) and Cu-O-O(hydrogen donor), respectively, in their order. Interatomic distances are given in Å and angles in degree.

directed towards Cu and a hydrogen-donor atoms. These sp^2 orbitals may extend within the plane normal to the basal plane of an anion and passing through the Se and O atoms, since in this conformation the oxygen atoms are bound most tightly to the Se atom by the additional formation of π -bonds. The p_x orbitals of oxygen atoms may then direct midway between the z - and x -(or y -) axes of Cu coordinate system and have only a negligible effect on the superexchange interaction between Cu atoms. The main paths of the superexchange interaction may thus involve an O-Se-O σ -bond, and the path Se-O-Cu may be weaker than the other two since the deviation of the Cu atom from the sp^2 lobe is the greatest for this oxygen atom (Fig. 5). The path Cu-O 1-Se-O 2-Cu...makes up a helical one-dimensional spin system along the crystallographic a axis. Though the difference between the three Se-O-Cu paths brings forth a linear magnetic system, the Se-O 3-Cu path may introduce a three-dimensional interaction between Cu(II) ions because the difference is not serious.

The magnetic susceptibility of $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ can be explained qualitatively on the refined crystal structure. Some of the parameters concerning the magnetic interactions were estimated phenomenologically. For a one-dimensional spin system there are two formalisms of magnetic susceptibility based on the Ising³⁰ and Heisenberg³¹ spins. The Ising model could not explain the temperature dependence of the susceptibility above 30 K, while the Heisenberg model gave a fair agreement. The g and J/k values giving the best fit are 2.00 and -33 K, respectively

(Fig. 4). On the other hand, the molecular field theory^{32,33} gives the relationships among θ , T_N , and J values. If a single J value is assumed for three O-Se-O paths, the observed θ gives $J/k = -31$ K, agreeing excellently with the value obtained by the Heisenberg model. The results also show that the difference in magnitude of superexchange interactions is very small. The ratio $|\theta|/T_N$ was estimated to be 3.0 using the above J value, which is comparable with the observed one (3.5). The g value of 2.00 seems smaller for a Cu(II) compound and the difference between two J/k values appears too small to give the magnetic behaviour of a linear spin system. An explanation for this may be as follows. Since each SeO_3^{2-} anion intermediates three copper(II) ions antiferromagnetically, the spin at each copper(II) cation is required to be "up" by the Cu'-O 1-Se-O 3-Cu interaction and at the same time to be "down" by the Cu''-O 2-Se-O 3-Cu interaction. The two interactions may be almost cancelled to give the observed features. The treatment is only approximate and the individual values cannot be taken to be accurate. Agreement as a whole indicates that the explanation is adequate.

In this mechanism the σ -bonds play an important role for the superexchange interaction. In $\text{Ba}_2\text{Cu}(\text{HCOO})_6 \cdot 4\text{H}_2\text{O}$ ³⁴ and a blue modification and a royal blue one of $\text{Cu}(\text{HCOO})_2$ ²⁶ the σ -bonds of an HCOO^- anion induce a ferromagnetic spin ordering and the π -bonds an antiferromagnetic one. If these schemes for formates are applied to the present system, $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ should be ferromagnetic, which is contrary to observation. In the nuclear spin-spin coupling in a high resolution NMR,³⁵ the sign of the multi-bond coupling is known largely to depend on the molecular conformation even in the saturated hydrocarbons, as well as on the number of intermediate bonds. It means that the spin-spin coupling of the magnetic ion depends on the species of the intervening oxo-anions and on their conformations in the crystal structure. In fact, the very simple model without any account for the conformation leads to the antiferromagnetic coupling of two Cu(II) spins through only the σ -bonds of SeO_3^{2-} anion. Thus, it is not astonishing that $\text{CuSeO}_3 \cdot 2\text{H}_2\text{O}$ shows antiferromagnetic behaviour without any superexchange interaction through π -bonds of the anion.

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